A Fast Algorithm for the Simulation of GCHP Systems

Louis Lamarche, PhD, PE

Benoit Beauchamp
Student Member ASHRAE

ABSTRACT

Annual hourly (or shorter) energy simulations are an important part of the design and analysis of ground-coupled heat pump systems. In order to evaluate the fluid temperature in the borehole of a geothermal heat pump system, most of the current models express the heat transfer rate as a sum of step changes in heat transfer rate. The borehole temperature is then computed as a superposition of the different contributions of each time step. The main difference between the different models lies in the way the step response is computed, whether a cylindrical heat source method, a line source method, or a tabulated numerical step response approach is used. Since all these methods are based on a convolution scheme, long time simulations are very time consuming since impulse response is recomputed at each time step. Many load aggregation algorithms have been proposed in order to reduce this computational time. In this paper, we present a new algorithm to evaluate the overall response, which is much faster than the classical convolution scheme.

INTRODUCTION

Ground-coupled heat pump (GCHP) systems are very effective in lowering the energy used to heat and cool residential and commercial buildings. The main factor that limits the growth of such systems is the initial cost of the borehole in the ground. As energy cost will increase, this cost will become relatively lower, and it is believed these systems will be very attractive in the near future. Still, the length of the ground exchanger will always be a very important factor, and a lot of research has been done in the design and analysis of this part of the GCHP system. Most of the models are based on the solution of the impulse response on a heat pulse in the borehole. The difference in the models is mostly in the way the heat conduction problem in the ground is solved and the way the interference problem between boreholes is treated A good survey of the different models is given by Yavuzturk (1999). Without going too much into the details of these methods, we may split these methods into two main approaches: analytical and numerical methods. In both cases some workers analyze only a one-dimensional transient problem in the radial direction, where $T(r, t)$ is sought in the field considered, whereas some models are based on the solution of the axisymmetric problem $T(r,z,t)$. A lot can be said about the validation of both approaches. For example, the radial problem does not give a steady-state solution; it has a logarithm singularity at infinity. One may argue about its validity after a long period of time (Eskilson 1987) if the load is constant. In the case of a symmetric annual load, this long-term effect should, however, not be so important. In any case, many design programs, such as the ones by Kavanaugh (1985) and Bernier (2001), use such a solution and give good results, as mentioned by Shonder et al. (1999).

Analytical methods are also given for the axisymmetric problem by Zeng et al. (2002), but most of them solve the radial problem, and the two major solutions used are the line source solution of Kelvin and the cylindrical heat source method. In both cases, they suffer from the fact that the solution is given in terms of a convolution solution, where each term has to be recomputed at each time step. This is the reason why the computing time is proportional to the square on the time interval. This precludes their use for a short time step (an hour or less) and/or for long period of time (a year or more). In order to reduce the computing time, Yavuzturk and Spitler (Yavuzturk 1999; Yavuzturk and Spitler 1999) proposed the concept of aggregation for loads that were applied a long time

Louis Lamarche is a professor and **Benoit Beauchamp** is a graduate student in the Department of Mechanical Engineering, École de Technologie Supérieure, Montréal, Québec, Canada.

before the actual time when the temperature is sought. They define a minimum hourly time history (MHTH) where no aggregation is done and also an aggregation period where a mean load for this aggregation period is computed and used in the simulations. This approach diminishes the simulation time by 90% for a year's simulation. Bernier et al. (2004) also proposed a multiple-load aggregation algorithm (MLAA) in order to cope with the same problem. In this paper, we present a very effective algorithm to solve the same heat conduction problem that is not history-dependent and is very efficient.

PROBLEM FORMULATION

The heat exchange problem in a buried vertical borehole can be formulated with respect to the schematic in Figure 1. If we neglect axial temperature variation, the basic problem is to find the temperature distribution $T(r,t)$ satisfying the heat conduction equation,

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}, \tag{1}$$

for the domain $r > r_b$, $t > 0$, and the following boundary conditions:

$$T(r,0) = T_o,\ -k\frac{\partial T}{\partial r}\bigg|_{r=r_b} = q_b''(t) = \frac{q_b'(t)}{2\pi r_b} \tag{2}$$

where q_b' is the heat flow per unit length q_b' is often referred to as the heat entering the borehole; here we keep the normal convention as the heat in the positive radial direction). In the case of step-function $q_b'(t) = q_o'\,u(t)$, the solution is well known and is given in the classic book of Carslaw and Jaeger (1947).

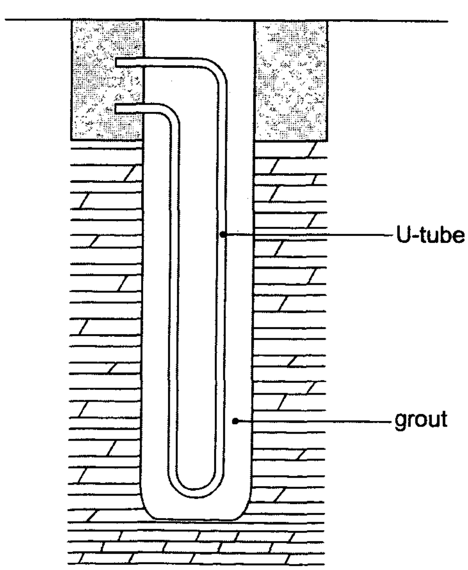

Figure 1 *Schematic of the borehole.*

$$(T(\tilde{r},\tilde{t}) - T_o) = \frac{q'_o}{k}\underbrace{\frac{1}{\pi^2}\int_o^\infty\left\{-\frac{e^{-z^2\tilde{t}}-1}{z^2[J_1^2(z)+Y_1^2(z)]}\right\}[J_o(\tilde{r}z)Y_1(z) - J_1(z)Y_o\tilde{r}z]dz}_{G(\tilde{r},\tilde{t})} \tag{3}$$

where $\tilde{r} = r/(r_b, \tilde{t}) = (\alpha t)/r_b^2$ = Fourier number.

This solution is known as the cylindrical heat source method (CHSM). Those using this solution for the analysis of GCHP systems (Kavanaugh 1985; Bernier 2001) use an analytical approximation of the G-function in their computation with the extension of arbitrary loads and the following expression:

$$T(\tilde{r},\tilde{t}) - T_o = \frac{1}{k}\sum_{i=1}^{N}(q_i - q_{i-1})G\left(\tilde{r}, \frac{\alpha(t-t_{i-1})}{r_b^2}\right) \tag{4}$$

In order to describe our new algorithm, we will solve again the heat conduction problem. Whereas Carslaw and Jaeger (1947) use the Laplace transform technique to present their solution, we will use Green's function formalism here. The solution of the general heat problem is well explained by Ozisik (1993):

$$\begin{aligned} T(r,t) = {} & \int_{r_b}^{\infty} f_i(\rho)H(r,\rho,t)\rho d\rho \\ & + \frac{\alpha}{k}\int_0^t q_b''(\tau)r_b H(r,r_b,t-\tau)d\tau \\ & + \frac{\alpha}{k}\int_0^t d\tau\int_{r_b}^{\infty}\dot{q}(\rho,\tau)H(r,\rho,t-\tau)\rho d\rho \end{aligned} \tag{5}$$

where $\dot{q}$ is the volumetric heat source distribution, $f_i(r)$ is the temperature distribution for $t = 0$, and $H(r, \rho, t-\tau)$ is what is known as the Green's function associated to the problem. The symbol G is normally associated with this function, but we will use H here in order not to confuse it with the cylindrical heat source function. Since, in our problem $\dot{q} = 0$ and with the change of variable, $\tilde{T} = T - T_o$, we have

$$\tilde{T}(r,t) = \frac{\alpha}{2\pi k}\int_0^t q_b(\tau)H(r,r_b,t-\tau)d\tau\,. \tag{6}$$

Following Ozisik (1993), we can find the Green's function by solving the associated problem with homogeneous boundary conditions:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{7}$$

for the domain $r > b$, $t > 0$, and the following boundary conditions:

$$T(r,0) = f_i(r),\ -k\frac{\partial T}{\partial r}\bigg|_{r=b} = 0 \tag{8}$$

Solving the problem and comparing with the Green's formalism solution,

$$T(r,t) = \int_{r_b}^{\infty} f_i(\rho) H(r,\rho,t)\rho d\rho \,, \tag{9}$$

we can get the associated Green's function by association. This associated problem can be solved by the Weber transform, but we will directly use the solution given by Cole (2000),

$$H(r, r_b, t-\tau) = \frac{1}{2\pi}\int_0^{\infty} e^{-\alpha\beta^2(t-\tau)} \frac{\beta R(r) R(r_b)}{[J_1^2(\beta r_b) + Y_1^2(\beta r_b)]} d\beta \,, \tag{10}$$

where

$$R(r) = J_o(\beta r) Y_1(\beta r_b) - J_1(\beta r_b) Y_o(\beta r) \,. \tag{11}$$

Inserting this solution into Equation 6, we get

$$\tilde{T}(r,t) = \frac{\alpha}{2\pi k}\int_0^t q_b'(\tau)\int_0^{\infty} e^{-\alpha\beta^2(t-\tau)} \frac{\beta R(r) R(r_b)}{[J_1^2(\beta r_b) + Y_1^2(\beta r_b)]} d\beta d\tau \,. \tag{12}$$

With Equation 11, we can simplify:

$$R(r_b) = J_o(\beta r_b) Y_1(\beta r_b) - J_1(\beta r_b) Y_o(\beta r_b) = \frac{-2}{\pi \beta r_b} \tag{13}$$

$$\tilde{T}(r,t) = \frac{-\alpha}{\pi^2 r_b k}\int_0^t q_b'(\tau)\int_0^{\infty} e^{-\alpha\beta^2(t-\tau)} \frac{R(r)}{[J_1^2(\beta r_b) + Y_1^2(\beta r_b)]} d\beta d\tau \tag{14}$$

If we apply a step change of heat, the response is:

$$\tilde{T}(r,t) = \frac{-q_0'\alpha}{\pi^2 r_b k}\int_0^{\infty} \frac{R(r)}{[J_1^2(\beta r_b) + Y_1^2(\beta r_b)]} d\beta \int_0^t e^{-\alpha\beta^2(t-\tau)} d\tau \tag{15}$$

$$\begin{aligned}\tilde{T}(r,t) &= \frac{q_o'}{k}\frac{1}{\pi^2}\int_0^{\infty} \frac{(e^{-z^2\tilde{t}} - 1)[J_o(\tilde{r}z)Y_1(z) - J_1(z)Y_o(\tilde{r}z)]}{z^2[J_1^2(z) + Y_1^2(z)]} dz \\ &= \frac{q_o'}{k} G(\tilde{r}, \tilde{t})\end{aligned} \tag{16}$$

where

$\tilde{r} = r/r_b$, $\tilde{t} = \alpha t / r_b^2$, and $z = \beta r_b$, which is the expected result.

NONHISTORY-DEPENDENT SCHEME

As mentioned in the introduction, the major problem with the convolution scheme is that, at each time step, the weighted G-function has to be recomputed, which results in an algorithm that is proportional to N^2, where N is the number of time steps. The idea of our new scheme is similar to the one of Greengard and Strain (1990) in the context of the boundary element method (BEM) for transient heat transfer problems. In their case, the problem was even greater, since they used, as is the usual case in BEM, the fundamental solution to Green's function as the kernel of the boundary integrals:

$$H(x, \xi, t, \tau) = \frac{1}{4\pi(t-\tau)} e^{\frac{-r^2}{4\alpha(t-\tau)}} \tag{17}$$

where $r = |x - \xi|$.

The difficulty arises from the coupling behavior of the Green's function as the space and time integration variables appear in the exponential term. In order to overcome this problem, they had to use some kind of Fourier expansion of the Green's function. In our particular GCHP problem, the integral solution is already in the form of a degenerate kernel since the space variable and the time variable are uncoupled. This facilitates the scheme.

Let's start with the general solution described in the previous section (Equation 14):

$$\tilde{T}(\tilde{r}, \tilde{t}) = \frac{-1}{\pi^2 k}\int_0^{\tilde{t}} q_b'(\tilde{t})\int_0^{\infty} e^{-z^2(\tilde{t}-\tilde{\tau})} \frac{J_o(\tilde{r}z)Y_1(z) - J_1(z)Y_o(\tilde{r}z)}{[J_1^2(z) + Y_1^2(z)]} dz d\tilde{\tau} \tag{18}$$

If we interchange the order of integration:

$$\tilde{T}(\tilde{r}, \tilde{t}) = \frac{-1}{\pi^2 k}\int_0^{\infty} \frac{J_o(\tilde{r}z)Y_1(z) - J_1(z)Y_o(\tilde{r}z)}{[J_1^2(z) + Y_1^2(z)]}\int_0^{\tilde{t}} q_b'(\tilde{\tau}) e^{-z^2(\tilde{t}-\tilde{\tau})} d\tilde{\tau} dz \tag{19}$$

In order to simplify the notation, we will make the change of variable:

$$v(\tilde{r}, z) = \frac{J_o(\tilde{r}z)Y_1(z) - J_1(z)Y_o(\tilde{r}z)}{[J_1^2(z) + Y_1^2(z)]} \tag{20}$$

Let us assume that the improper integrals converge correctly and that we can keep a finite number of terms in the approximation of the integral:

$$\begin{aligned}\tilde{T}(\tilde{r}, \tilde{t}) &= \frac{-1}{\pi^2 k}\sum_{n=1}^{N} v_n(\tilde{r}, z)\int_0^{\tilde{t}} q_b'(\tilde{\tau}) e^{-z^2(\tilde{t}-\tilde{\tau})} d\tilde{\tau} \Delta z_n \\ &= \frac{1}{\pi^2 k}\sum_{n=1}^{N} F_n(\tilde{t}) \Delta z_n\end{aligned} \tag{21}$$

At the next time interval, we can evaluate the new temperature distribution:

$$\begin{aligned}\tilde{T}[\tilde{r}, (\tilde{t} + \Delta\tilde{t})] &= \frac{-1}{\pi^2 k}\sum_{n=1}^{N} v_n(\tilde{r}, z)\Big[\int_0^{\tilde{t}} q_b'(\tilde{\tau}) e^{-z^2(\tilde{t}+\Delta\tilde{t}-\tilde{\tau})} d\tilde{\tau} \\ &\quad + \int_0^{\tilde{t}+\Delta\tilde{t}} q_b'(\tilde{\tau}) e^{-z^2(\tilde{t}+\Delta\tilde{t}-\tilde{\tau})} d\tilde{\tau}\Big]\Delta z_n\end{aligned} \tag{22}$$